Reply

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The original comments on the paper by Lary and Balluch (1993, hereafter LB) raised three main and one minor criticism. The main criticisms were that

1) the direct beam has been calculated wrongly in LB, that is, without a Chapman function,
2) the source function as defined in Eq. (6) in LB omits the direct solar irradiance, and
3) Eq. (1) of LB, the basic equation for the whole model introduced in LB, is wrongfully applied in the context discussed in LB.

The minor criticism concerned the drawing in Fig. 1 of LB, which is said to be out of proportion.

In the revised version of the comments, however, the main emphasis of the criticism was put on the azimuthal asymmetry, which stems from the fact that the solar zenith angle on a spherical earth is different on different latitudes and longitudes at a given instant in time. Since scattering involves light coming from areas other than just the vertical column of air under consideration, in principle all other vertical columns above the surface of the earth on different longitudes and latitudes have to be included in the calculation. That this is the argument raised in the comments can be seen from Fig. 1 of the comments. The center of the coordinate system refers to the center of the earth. If r_0 refers to the vertical column of air under consideration, then, clearly, the line marked r intersects the surface of the earth on a place different from the line marked r_0 and therefore refers to a different longitude and latitude. Indeed, on different longitudes and latitudes at a given instant in time the solar zenith angle differs because the local time and local season differ. The effect of this variation of zenith angles on scattered light from neighboring areas around the vertical column of air under consideration has been neglected by LB in their calculations.

In the following, each of the major criticisms of the original version will be discussed and argued to be invalid. The assumption of neglecting the variation of solar zenith angles to calculate the effect of scattering from different longitudes and latitudes will be discussed, and the errors made due to that assumption will be estimated. Finally, some remarks will be given on the minor criticism.

To prove or disprove a statement one has to start from a position on whose validity both parties agree. From the nature of the original comments one could conclude that the authors generally agree that the basic formulas given in Dahlback and Stammes (1991, hereafter DS) are correct. Therefore, it will suffice for the purpose of this reply to prove mathematically the equivalence of the calculation of the direct beam, the calculation of the source function, and the formulation of the basic equation in LB compared to DS. However, in the revised version of the comments the authors claim that DS also did not include the azimuthal variation due to the variation of the solar zenith angle with latitude and longitude for the calculation of the contribution of scattering. This will therefore be discussed separately.

1) The calculation of the direct beam

The direct beam in DS is calculated by

$$I_s(r_p) = I_0\, e^{-ch(r_p,\mu_0)}, \tag{1}$$

where I_0 is the solar irradiance at the top of the atmosphere, r_p the distance to the center of the earth, and $\mu_0 = \cos\theta_0$ the cosine of the angle to the outward normal of the direct solar beam, that is, the cosine of the solar zenith angle at r_p. The Chapman function is given in Eq. (9) in DS. Given that the vertical optical depth is $\Delta\tau_j = \sigma_e(j)\Delta h_j$, where $\sigma_e(j)$ is the extinction coefficient averaged over the jth layer bounded by r_j and r_{j+1},

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and using Eq. (B1) of the appendix B in DS, we arrive at

$$ch(r_p, \mu_0) = \sum_{j=1}^{p} \sigma_e(j) \left(\sqrt{r_j^2 - r_p^2 \sin^2\theta_0} - \sqrt{r_{j+1}^2 - r_p^2 \sin^2\theta_0} \right) \quad (2)$$

for the case of $\theta_0 \leq 90°$.

In LB the direct beam is calculated as follows. Equation (4) of LB with $S = 0$ is integrated along the characteristic $P = r_j \sin\theta_j = r_p \sin\theta_0$ (P of LB corresponds to the distance OG in appendix B of DS). Therefore,

$$I_s = I_0 e^{-\tau_p}, \quad (3)$$

where τ_p is given by Eq. (5) of LB as

$$\tau_p = \sum_{j=1}^{p} \sigma_e(j) (\mu_j r_j - \mu_{j+1} r_{j+1}). \quad (4)$$

Since

$$\mu_j = \cos\theta_j = \sqrt{1 - \sin^2\theta_j} = \left(1 - \frac{P^2}{r_j^2}\right)^{1/2} \quad (5)$$

and $P = r_p \sin\theta_0$, the expressions in Eqs. (2) and (4) are equivalent. Therefore, LB and DS calculate the direct beam with analytically equivalent numerical expressions [Eqs. (1) and (3)]. The same applies, as an easy calculation shows, to the case $\theta_0 > 90°$ as well.

2) *The definition of the source function*

In LB the definition of the source function as given in Eq. (6) of LB for the case of isotropic scattering and neglecting the Planck function reads

$$S(r) = \frac{\sigma_s}{\sigma_e} \frac{1}{2} \int_{-1}^{1} I(r, \mu) d\mu. \quad (6)$$

The total intensity $I(r, \mu)$ can be split into the solar part I_s and the diffusive part I_d. Then Eq. (6) reads

$$S(r) = \frac{\sigma_s}{\sigma_e} \frac{1}{2} \int_{-1}^{1} \left(I_s \frac{\delta(\mu - \mu_0)}{2\pi} + I_d \right) d\mu. \quad (7)$$

Integrating the part with the δ distribution we arrive at

$$S(r) = \frac{\sigma_s}{\sigma_e} \left(\frac{I_s}{4\pi} + \frac{1}{2} \int_{-1}^{1} I_d \, d\mu \right). \quad (8)$$

This is equivalent to the definition of the source function as given in Eq. (7b) of DS for the case of isotropic scattering. Therefore, again, LB and DS have totally equivalent formulations for the definitions of their source functions.

3) *The basic radiation transport equation*

The criticized equation (1) of LB is clearly equivalent to Eq. (7a) of DS for the case of azimuthal symmetry, that is, when the azimuthal terms can be neglected. Indeed, if we neglect the variation of the solar zenith angle with different latitudes and longitudes for the scattered light, the problem of isotropic scattering in spherical geometry becomes azimuthally independent simply because there is no physical process involved that depends on the azimuth. The only azimuthal dependence in this problem is introduced by the scattering phase function, which is constant and 1 in the isotropic case. The direct beam reduces to an azimuthally symmetric source for the intensity in each layer and therefore does not break the azimuthal asymmetry as well.

As mentioned above, for calculating the heating rates in one vertical column of air LB neglected the variation of the solar zenith angle with different latitudes and longitudes for the scattered light. This was done because

- including this effect would increase the computational costs enormously since this involves extending the problem for at least two further dimensions compared to the case of isotropic scattering, and
- the errors made due to that neglect are very small for the calculation of heating rates, being generally below 0.1% in areas of significant heating rates and less than 0.0035 K/day in absolute terms. Actually, as can be seen for example in Balluch (1994), neglecting the variability of the phase function for scattering on air molecules, that is, the assumption of isotropic scattering, leads to much larger errors of up to 20% or 0.35 K/day in absolute terms. Even neglecting the scattering on aerosols has a larger effect on the heating rates, namely, up to 6% in the troposphere and around 1.5% at a height of 40 km.

In the revised comments the main emphasis is put on the azimuthal variation of the intensity due to the variation of the solar zenith angle with different latitudes and longitudes for the calculation of the scattered light. This effect, however interesting in its own right, does not necessarily indicate an effect of similar size on the heating rate. Scattering on aerosols, for example, can lead to azimuthal asymmetries of 60% and more in certain wavelength bands, but the overall effect on the heating rate is not particularly noteworthy (see Balluch 1994).

Figures 2 and 3 in the comments show the maximum azimuthal variation in specific angles in a small wavelength band at a solar zenith angle of $\theta_0 = 85°$. Presumably this setup was chosen to show the largest azimuthal variation. However, the small ratio of the specific intensity over the solar irradiance of less than 0.004 indicates already the marginal effect this might have on heating rates.

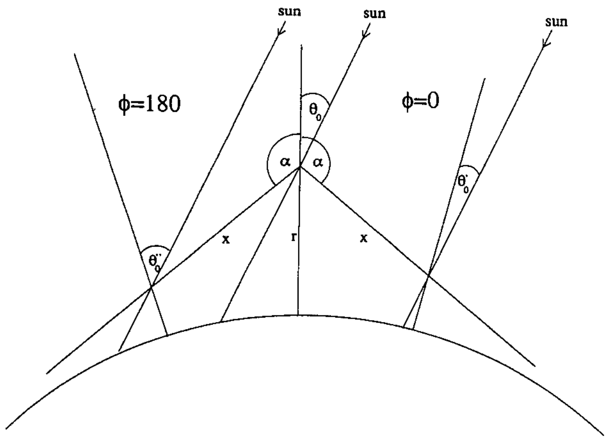

FIG. 1. Geometry for the calculation to estimate to first-order scattering the effects of the variation of solar zenith angles. The solar zenith angles satisfy $\theta_0' < \theta_0 < \theta_0''$.

If we perform the following calculation we can estimate the errors in the heating rate including first-order scattering, which occur due to neglecting the variation of the solar zenith angle with different latitudes and longitudes for the calculation of the scattered light. At each point of the vertical column of air under consideration we choose a set of angles α. Each of these angles α gives one line of sight with azimuth $\phi = 0°$ and one line of sight with azimuth $\phi = 180°$ through the atmosphere toward each point r on the vertical column of air, where r is the distance to the center of the earth from this point. The geometry of the problem is shown in Fig. 1. We can then calculate the direct solar irradiance on a set of discrete points along a given line of sight, taking into account that the respective solar zenith angles for azimuth $\phi = 0°$, θ_0' and for $\phi = 180°$, θ_0'' are different from the one on the original vertical column of air under consideration, θ_0 (see Fig. 1). We can then integrate the contribution of first-order scattering to the source function along these lines of sight by solving

$$I(\tau) = e^{-\tau} \int_0^\tau S(x)\, e^x\, dx. \tag{9}$$

Equation (9) is the general solution of the three-dimensional equation of radiative transfer along each line of sight with boundary condition $I(0) = 0$. Here τ is the optical depth along the line of sight parameterized by x. As stated above, for this error estimate the source function S was approximated by first-order scattering only:

$$S(x) = \frac{\sigma_s(x)}{\sigma_e(x)} J_{\mathrm{dir}}(x), \tag{10}$$

where $J_{\mathrm{dir}}(x)$ is the direct solar irradiance on the point in the atmosphere characterized by the parameter x on the line of sight α toward the point r on the vertical column of air under consideration. Here $J_{\mathrm{dir}}(x)$ differs with differing (x, α, r) due to differing vertical heights and due to differing longitudes and latitudes, that is, due to differing solar zenith angles.

In that way we can calculate contributions of the azimuth angles $\phi = 0°$ and $\phi = 180°$ to the local radiation flux. These can be compared with the contributions of the same azimuth angles where the variation of the solar zenith angle was neglected. The latter can be done by simply calculating the vertical height $\hat{r}$ corresponding to the parameter set (x, α, r) and setting $J_{\mathrm{dir}}(x) = \hat{J}_{\mathrm{dir}}(\hat{r})$, where $\hat{J}_{\mathrm{dir}}(\hat{r})$ is the direct solar irradiance at the height $\hat{r}$ on the vertical column of air under consideration. For the results presented here, up to 96 values for x on each line of sight, depending on the length of this line, and 32 values of α were chosen for each vertical height r on the vertical column of air under consideration. The ozone profile, the vertical discretization, and the wavelength distribution were chosen as in LB.

Assuming that the case $\phi = 0°$ approximates the area of $\phi < 90°$ and that the case $\phi = 180°$ approximates the area of $\phi > 90°$ we can calculate a heating rate for first-order scattering at each point on the vertical column of air under consideration (a) with varying solar zenith angle and (b) with constant solar zenith angle. The absolute difference between the two is plotted in Fig. 2 for a solar zenith angle on the vertical column of air under consideration of $\theta_0 = 85°$. This compares to the situation depicted in Figs. 2 and 3 in the comments. We can see now that the absolute difference in heating rates is less than 0.0021 K/day. For larger as

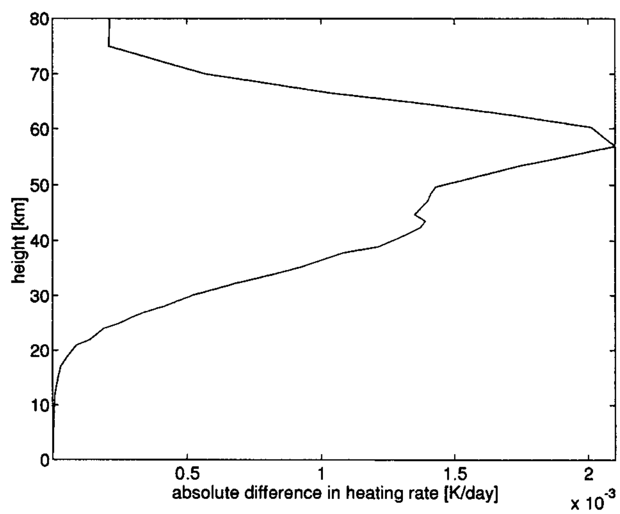

FIG. 2. The differences in the heating rates including only first-order scattering between a calculation made using a constant solar zenith angle minus a calculation made using varying solar zenith angles. The solar zenith angle on the column of air under consideration is $\theta_0 = 85°$. The differences are everywhere less than 0.0021 K day^{-1}.

well as for smaller solar zenith angles this absolute difference in heating rates actually decreases, confirming that the authors of the comments have chosen the case of maximum discrepancy in their Figs. 2 and 3. The relative difference for the case of $\theta_0 = 85°$ (see Fig. 3) is less than 0.1% everywhere. However, for very large solar zenith angles $\theta_0 > 92°$ the difference in heating rates reaches a higher albeit negative maximum of -0.0035 K/day between 40 and 60 km, which amounts to 0.4% of the total heating rate.

For a solar zenith angle of $\theta_0 = 0°$, that is, the overhead sun, the maximum difference in the heating rate is 0.00002 K/day; for $\theta_0 = 70°$ the maximum difference rises up to 0.00007 K/day; for $\theta_0 = 90°$ the maximum difference is 0.0013 K/day; for $\theta_0 = 92°$ the maximum difference is -0.0035 K/day; and for $\theta_0 = 96°$ the maximum difference is -0.0022 K/day. It is clear that these values are much below any relevance, and therefore the effect on heating rates of varying solar zenith angles with latitude and longitude for the calculation of the scattering can be safely ignored.

What remains is a brief reply to the minor criticism. It is true that Fig. 1 of LB is out of proportion, but in real proportions the drawing could not possibly serve its purpose to clarify which areas of the atmosphere experience solar radiation at zenith angles greater than 90 degrees. By the way, DS used a very similar picture to clarify the geometrical situation of the direct beam at solar zenith angles greater than 90 degrees (Fig. B1 in appendix B of DS). In fact, this figure (B1) of DS is nearly as much out of proportion as Fig. 1 of LB. The danger that Fig. 1 of LB exaggerates the effect of the sun at solar zenith angles greater than 90 degrees is counterbalanced by the fact that the whole paper is committed to estimate exactly that effect. An examination of the figures showing the results of the calculation would show qualitatively and quantitatively how large this effect really is.

In view of the aforementioned arguments and proofs, one can conclude that all the criticisms raised in the comments are unjustified and the results presented in the LB paper are correct. Lary and Balluch (1993) did neglect the variation of the solar zenith angle with varying latitude and longitude for the scattering calculation, but this assumption leads to errors less than 0.1% in the heating rate. Assuming isotropic scattering leads to much larger errors of up to 20%. Failure to include scattering on aerosols in the calculation has a much greater effect on the heating rate.

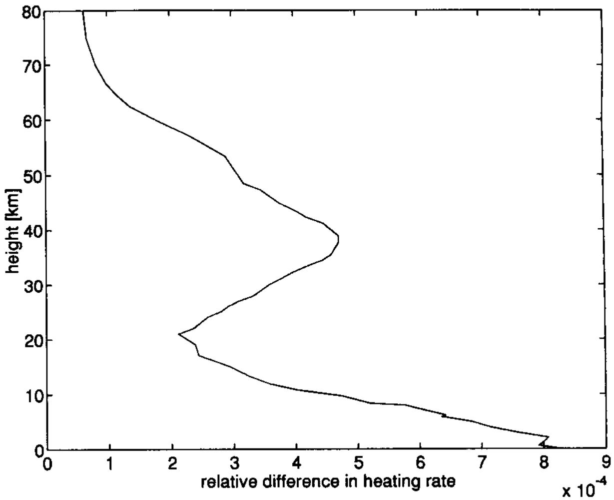

FIG. 3. The relative differences in the heating rate including only first-order scattering between a calculation made using a constant solar zenith angle minus a calculation made using varying solar zenith angles. The solar zenith angle on the column of air under consideration is $\theta_0 = 85°$. The relative differences are everywhere less than 0.1%.

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